Third. That they participate in all the changes the mesentery undergoes.

Fourth. That if an organ is developed in the mesentery or from the gut, it obtains, part at least, of its vascular supply from the vessels of the mesentery, or from those of that part of the gut from which it sprung.

These principles have, in this paper, only been applied to the development of the blood-vessels of the human alimentary canal, but they seem so simple, and the probability of their truth appears so great, that it may be anticipated that they will be found of very much wider application.

"The Influence of Stress and Strain on the Physical Properties of Matter.* Part I. Moduli of Elasticity—*continued.* Relations between Moduli of Elasticity, Thermal Capacity, and other Physical Constants." By HERBERT TOMLINSON, B.A. Communicated by Professor W. GRYLLS ADAMS, M.A., F.R.S. Received May 28, 1884. Read June 19.

It has been proved by Wertheim,† whose results have been verified by myself,‡ that if e be taken to denote "Young's Modulus," and α the mean distance between the centres of any two adjacent molecules of a solid body, $e \times \alpha^7$ is, in the case of most metals, approximately a constant. Poisson, regarding a body as an assemblage of molecules very small as compared with the distance between them, bound to one another by an attractive force, and kept at a distance by the repulsion due to heat, was led to the formula—

$$\frac{N}{\delta} = \frac{2\pi}{3} \sum_{r=\alpha}^{r=\infty} \frac{r^5}{\alpha^5} \frac{d\frac{1}{r}fr}{dr}.$$

In this formula N is a constant force perpendicular to the surface of the body, δ the corresponding linear expansion or contraction produced, r the radius of activity of a molecule, α the mean distance between two adjacent molecules, and fr the function by which the law of the resultant molecular force is expressed. It follows, therefore, that if $e \times \alpha^7$ is a constant, fr must decrease in the inverse

* The original title of the paper has been altered to the above as being more exact in expression.

† "Ann. de Chimie," 1884, tom. xii.

‡ "Phil. Trans.," Part I, 1883, p. 32.

ratio of the fifth power of the distance. Now, Clerk Maxwell in his valuable researches on the viscosity of gases arrived at the important result that the viscosity of a gas is independent of its density, and proportional to the temperature measured from the absolute zero of the air-thermometer.* Maxwell points out that the constancy of the viscosity of a gas for all changes of density when the temperature is constant is a result of the "Dynamical Theory of Gases," whatever hypothesis we adopt as to the mode of action between the molecules when they come near one another; but that if the viscosity be as the first power of the absolute temperature, then in the dynamical theory, which is framed to explain the facts, we must assume that the force between two molecules is proportional inversely to the *fifth power of the distance between them.*† It is true that we cannot regard the distance between two consecutive molecules of a solid as large compared with the diameter of the molecules; indeed, we must, on the contrary, assume the molecules to be nearly touching each other; consequently Poisson's formula would require modification before it could be strictly adapted to the case in point. It is equally true that the equation $e \times \alpha^7 =$ a constant cannot possibly hold good for all temperatures, inasmuch as the value of e decreases more with rise of temperature than α^7 increases, and that consequently the function fr must contain the temperature. Nevertheless, the coincidence of the results as regards the law of force between two molecules obtained by Wertheim and Maxwell in quite different ways, seemed to warrant me in making more extended inquiries in this direction, and with this object I set about making a most careful determination of the thermal capacity of each of the different metals which had been used by me in the earlier portion of this inquiry. My object in doing so will be seen from what follows:—

If we denote the atomic mass by A, the density by Δ, the thermal capacity per unit mass by C_m, and the thermal capacity per unit volume by C_v, we have the following relations:—

$$C_m \times A = \text{a constant};$$

$$C_v = \Delta \times C_m;$$

$$e \times \alpha^7 = \text{a constant};$$

$$\alpha \propto \left(\frac{A}{\Delta}\right)^{\frac{1}{3}}.$$

* "Phil. Trans.," 1866, vol. 126, Part I.

† I find that Maxwell in a later memoir ("Phil. Trans.," 1879, Part I) expresses an opinion that the balance of evidence is against this law. On the contrary, the results obtained by Heen ("Bulletins de l'Académie Royale de Belgique," 3e Série, t. iv, 1882), seem to prove most conclusively that for liquids and solids of a *definite constitution*, as shown by their thermal expansibility at different temperatures, the

From these relations we obtain—

$$\frac{e}{C_v^{\frac{7}{3}}} = \text{a constant},$$

or that the cube of "Young's Modulus" varies as the seventh power of the thermal capacity per unit volume.

Description of Apparatus and Mode of Experimenting.

Thermometers.—Three thermometers were employed; these were made by Casella, and after having been filled were kept by myself for a year; they were then sent back to the maker to be graduated, and a month afterwards their graduations were tested at Kew, and were found to accord very accurately with the Kew standards throughout the whole of their ranges, which were in the three instruments from −5° C. to 30° C., from 30° C. to 65° C., and from 65° C. to 105° C. respectively. The graduations were made to one-tenth of a degree centigrade, and were of such a length that it was easy to estimate to one-hundredth of a degree.

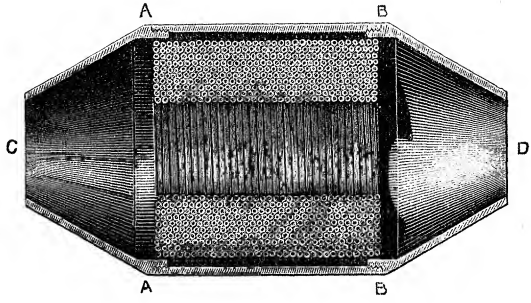
Calorimeter.—The calorimeter was of the well-known kind introduced by Regnault, and the silver coatings of both the inner and outer copper vessels of the instrument, which was new, were throughout the experiments kept uniformly bright. The mass of the inner vessel of the calorimeter was 133 grams, and the water equivalent of it, the stirrer and thermometer, was determined from the mean of several experiments made by introducing into the vessel known masses of water at different temperatures to be 138 grams, whilst the water equivalent, as calculated from the mass and thermal capacity of the vessel stirrer and thermometer, was, on the assumption of the heat being absorbed uniformly throughout the vessel, 140 grams. The probable error of an observation of the thermal capacity of a wire resulting from error in the determination of the water equivalent would in no case exceed 0·06 per cent.

Mode of Adjusting the Wires.

The wires, which had been previously well annealed, were wound round a steel rod, and so made into coils of length about 2 inches, inner diameter $\frac{3}{4}$ inch, and outer diameter $1\frac{3}{4}$ inches; the rod was then withdrawn, and the coil inserted into a thin brass envelope. The envelope (figs. 1, 2) consisted of a hollow cylinder AB, 2 inches in length and 2 inches in diameter, terminated at both ends by a truncated cone. The closed conical end BDB could be unscrewed so as to receive the coil of wire, and was, after the insertion of the coil,

force does vary inversely as the fifth power of the distance. According to M. Heen, the force is inversely as the seventh power of the distance, but it would seem to follow from Poisson's mathematics that the law should be really the one previously indicated.

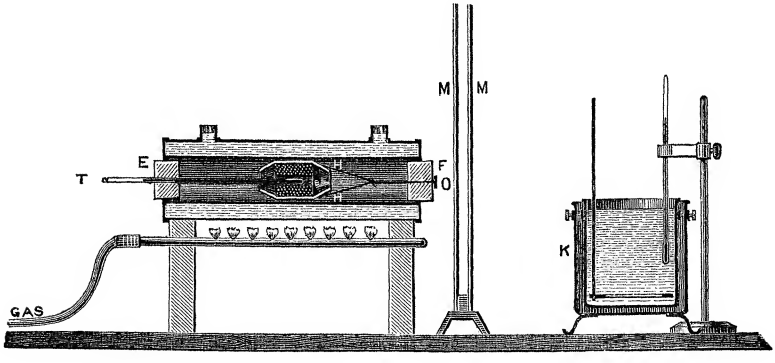
FIG. 1.



screwed on again; whilst at the other extremity of the envelope was an aperture, C, $\frac{3}{4}$ inch in diameter, through which a thermometer could be inserted so that its bulb would lie along the axis and in the centre of the coil of wire.

The envelope served a double purpose, as it not only enabled compensation to be made in the manner presently to be described for loss of heat during the transference of the wire from the air-chamber to the calorimeter, but also was of use in distributing the heat uniformly throughout the coil, which was not the case when no such precaution

FIG. 2.



was taken.* The thermometer was placed in the air-chamber in a position which was nearly horizontal, but slightly slanting upwards so as to prevent the column of mercury in the thermometer tube from being broken when the temperature was falling. As soon as the coil of wire had attained the desired temperature, the calorimeter was

* In some preliminary experiments made without the envelope I frequently found a difference of quite 1° between the temperature of the centre of the coil and that of one end, but with the envelope there was, after a certain interval of heating, no sensible difference of temperature.

placed close to the end F of the air-chamber, and the double screen MM having been first removed the cork F was pulled out, and after it, through the intervention of the thread, the brass envelope with the contained wire, which were speedily dropped into the calorimeter. After the water in the calorimeter and its contents had been stirred for three minutes, the temperature of the mixture was determined to within $\frac{1}{100}$° C. by means of the first of the previously-mentioned thermometers, care having been taken by means of the erection of certain marks to avoid parallax in the reading. After the first reading had been taken two others ensued after successive intervals of three minutes each, so that correction might be made for loss of heat resulting from radiation; this loss, however, being almost compensated by placing in the calorimeter water at a temperature which previous rough calculation had shown would be as much below the temperature of the room as the temperature of the mixture would be above it.

Preliminary observations were made for the purpose of ascertaining the rise of temperature which would be caused by immersing the heated envelope only in the 450 grams of distilled water used in the calorimeter, and the mean of eighteen observations in which the envelope alone was raised to 60° C. and then plunged into water at about 20° C., showed that the average rise of temperature per degree of fall of temperature of the brass envelope between 60° C. and 20° C. was 0·01150° C. Similarly eighteen observations between 100° C. and 20° C. showed a rise of 0·01114° C. to be produced per degree of fall of temperature of the envelope. If the thermal capacity of the brass of which the envelope was formed be assumed to be 0·0924 between 20° C. and 60° C.,* the calculated rise of temperature, supposing that no heat is lost in the transference of the envelope from the air-chamber to the calorimeter, would be 0·0137; so that, as might be expected from the large surface of the envelope as compared with its mass, the loss of heat during the transference is very appreciable, and, as also might be expected, the loss between 100° C. and 20° C. is greater than that between 60° C. and 20° C. It will be sufficient, perhaps, to record the eighteen observations between 20° C. and 100° C.

* My subsequent experiments would prove that this would be very nearly the case.

Experiment I.—The brass envelope heated to 100° and afterwards plunged into 450 grams of water contained in the calorimeter.

Number of observation.	Average rise of temperature of the water and calorimeter per degree of fall of temperature of the envelope.	Departure from the mean value.
1	·01138	+ ·000024
2	·01115	+ ·000001
3	·01106	− ·000008
4	·01121	+ ·000007
5	·01103	− ·000011
6	·01121	+ ·000007
7	·01102	− ·000012
8	·01089	− ·000025
9	·01125	+ ·000011
10	·01136	+ ·000022
11	·01091	− ·000023
12	·01130	+ ·000016
13	·01141	+ ·000027
14	·01096	− ·000018
15	·01115	+ ·000001
16	·01112	− ·000002
17	·01105	− ·000009
18	·01107	− ·000007
Mean	·01114	

The probable error of the mean value is not more than 0·22 per cent., and the probable error of the mean value of the other set of eighteen observations was not more than 0·16 per cent. Experiment II furnishes a fair sample of the accuracy of the work.

Experiment II.—German-silver (1)* well annealed, weighing 121·08 grams.

No. of Observation.	Temperature of the envelope and contained wire before mixing.	Temperature of mixture uncorrected.	Correction to be applied for effects of radiation.	Temperature of mixture corrected.	Rise of temperature caused in the water, calorimeter, &c., by envelope and contained wire.	Calculated rise of temperature caused by the envelope only.	Rise of temperature caused by the wire only.	Mean thermal capacity between 20° C. and 60° C. The thermal capacity of water at 20° C. = 1.	Departure from the mean value of all the observations.
1.....	60·60	19·820	† − ·0225	19·7975	1·2875	·4686	·8189	·09366	− ·00058
2.....	59·43	20·205	+ ·0375	20·2425	1·2425	·4507	·7918	·09416	− ·00008
3.....	61·93	22·195	+ ·0375	22·2325	1·2725	·4566	·8159	·09578	+ ·00154
4.....	61·58	21·605	·0000	21·605	1·2850	·4597	·8253	·09624	+ ·00200
5.....	61·70	21·845	+ ·0150	21·8600	1·2600	·4582	·8018	·09380	− ·00044
6..... ...	60·88	20·505	− ·0075	20·5125	1·2775	·4644	·8131	·09385	− ·00039
7.....	56·62	20·140	+ ·0400	20·1800	1·1400	·4191	·7209	·09220	− ·00204

* A number attached to the name of a metal refers to a wire with a corresponding number in a portion of the paper already published ("Phil. Trans.," Part I, 1883).

† A minus sign in the column shows that the water in the calorimeter had been cooled down a little too much before the mixing took place, and that hence the water gained in temperature by radiation.

The mean value of all the observations on the mean thermal capacity of German-silver between 20° C. and 60° C. is 0·09424, but this number requires to be slightly corrected if we are to assume the thermal capacity of water at 0° C. to be unity. According to Regnault,* the thermal capacity of water at t° C. $= 1 + 0{\cdot}00004t + 0{\cdot}0000009t^2$, and hence the thermal capacity at 20° C. = 1·00116. Again the apparent weight of water in air was taken instead of the weight *in vacuo*. Now the correction of the apparent weight W of a mass of water when weighed with brass weights amounts to $W \cdot 0{\cdot}0012 (1 - \frac{1}{8{\cdot}4}) = W \cdot 0{\cdot}00106$. On the whole, therefore, we must take 0·09424 × 1·00222, *i.e.*, 0·09445, as the corrected thermal capacity. A similar correction was applied in the case of all the other observations. No correction was made in consideration of the fact that the whole of the stem of the thermometer employed in the calorimeter was not immersed in the source of heat, as according to Kopp, if N be the number of degrees extant from the source of heat, T the observed temperature, and t the atmospheric temperature, α the correction to be applied,

$$\alpha = N(T - t) \times {\cdot}0001545.\dagger$$

Now if we assume the temperature of the air to be 20° C., and the temperature of the water before and after the immersion of the heated wire to be 18° C. and 22° C. respectively (this representing rather an extreme case), the correction would only introduce an alteration in the value of the thermal capacity of 0·015 per cent. With the thermometer used in the heated air-chamber a correction, not however exceeding in any case 0·3 per cent., was, when necessary, applied by the aid of Kopp's formula.

In Table I are given the results obtained with the different metals, and in Table II a comparison of these results with those obtained by other experimenters.

* "Mém. Acad. Sciences," xxi, p. 729.

† Recent investigations have shown that this formula can only be regarded as roughly approximate, but the approximation is quite sufficient for the present purpose.

Table I.

Name of metal.	Mean thermal capacity per unit mass between 20° C. and 60° C. Thermal capacity of water at 0° C. −1.	Number of observations between 20° C. and 60° C.	Probable error per cent.	Mean thermal capacity per unit mass between 20° C. and 100° C. Thermal capacity of water at 0° C. =1.	Number of observations between 20° C. and 100° C.	Probable error per cent.	Formulæ for the number of thermal units required to raise the temperature of unit mass from 0° C. to t° C.	Formulæ for the thermal capacity per unit mass at the temperature of t° C.
Aluminium	·21620	10	0·56	·22080	8	0·34	$\cdot20700t + \cdot0001152t^2$	$\cdot20700 + \cdot0002304t$
Iron.....	·11162	5	0·81	·11442	5	0·61	$\cdot10601t + \cdot0000701t^2$	$\cdot10601 + \cdot0001402t$
German-silver	·09445	7	0·48	·09477	3	0·19	$\cdot09413t + \cdot0000053t^2$	$\cdot09413 + \cdot0000106t$
Zinc	·09308	6	1·20	·09457	3	0·13	$\cdot09009t + \cdot0000374t^2$	$\cdot09009 + \cdot0000748t$
Copper	·09267	5	0·24	·09396	4	0·35	$\cdot09008t + \cdot0000475t^2$	$\cdot09008 + \cdot0000648t$
Silver.....	·05640	3	0·38	·05727	3	0·83	$\cdot05466t + \cdot0000218t^2$	$\cdot05466 + \cdot0000436t$
Tin.....	·05518	5	0·96	·05662	6	0·46	$\cdot05231t + \cdot0000361t^2$	$\cdot05231 + \cdot0000722t$
Platinum-silver	·04836	11	0·87	·04891	6	0·57	$\cdot04726t + \cdot0000138t^2$	$\cdot04726 + \cdot0000276t$
Platinum	·03248	5	0·69	·03273	2	0·39	$\cdot03198t + \cdot0000063t^2$	$\cdot03198 + \cdot0000125t$
Lead	·03120	3	0·74	·03181	6	1·00	$\cdot02998t + \cdot0000153t^2$	$\cdot02998 - \cdot0000306t$

Table II.

Observer.	Metal.	Formulæ for the thermal capacity per unit mass at $t°$ C.	Mean thermal capacity per unit mass between 0° C. and 100° C.	Departure per cent. from the mean of the values obtained by other observers.
Tomlinson.....	Aluminium	·20700 + ·0002304t	·2185	
Bède.....	Iron	·1040 + ·000144t	·1112	Mean. ·1116	1·4 +
Regnault		.. ..	·1138	
Dulong & Petit		.. ..	·1098	
Tomlinson.....		·10601 + ·000140t	·1130	
Tomlinson.....	German-silver	·09413 + ·0000106t	·09466	
Bède.....	Zinc	·08595 + ·000084t	·09015	Mean. ·09280	1·1 +
Regnault		.. ..	·09555	
Dulong & Petit		.. ..	·09270	
Tomlinson.....		·09009 + ·000075t	·09383	
Bède.....	Copper	·0892 + ·000065t	·09250	Mean. ·09418	0·9 −
Regnault		.. ..	·09515	
Dulong & Petit		.. ..	·09490	
Tomlinson.....		·09008 + ·000032t	·09332	
Dulong & Petit	Silver	·0530 + ·000027t	·05570	Mean. ·05685	0·0
Regnault		.. ..	·05701	
Tomlinson.....		·05466 + ·000044t	·05684	
Bède.....	Tin	·0512 + ·000063t	·05440	Mean. ·05531	1·1 +
Regnault		.. ..	·05623	
Tomlinson.....		·05231 + ·000072t	·05591	
Tomlinson.....	Platinum-silver	·04726 + ·000028t	·04864	
Violle.....	Platinum	·0317 + ·000012t	·03230	Mean. ·03274	0·4 −
Regnault		.. ..	·03243	
Pouillet.....		.. ..	·03350	
Dulong & Petit		.. ..	·03550*	
Tomlinson.....		·03198 + ·000013t	·03262	
Bède.....	Lead	·0283 + ·000036t	·03010	Mean. ·03075	2·5 +
Regnault		.. ..	·03140	
Tomlinson.....		·02998 + ·000031t	·03151	

* I have rejected this number from the mean, as it is considerably in excess of those of the other observers, and is, moreover, exactly equal to that obtained by Dulong and Petit between 0° C. and 300° C., a result not in accordance with the experiments of Pouillet, Violle, or myself, who have all found the thermal capacity of platinum to rise with the temperature.

Remarks on Tables I and II.

From the last column of Table I we learn that the thermal capacity of all the metals examined increases with the temperature, a result which we find confirmed by the observations of other experimenters, recorded in the third column of Table II.*

The last column of Table II shows that the results obtained by me are on the whole slightly, but decidedly, *greater* than those of the means of the other experimenters, and perhaps this *may* be accounted for by the fact that with me the loss of heat in the transfer from the hot air-chamber to the cold water is *entirely* compensated.

The thermal capacities of the alloys platinum-silver and German-silver are almost exactly the same as those calculated from the proportions of their components; thus the calculated thermal capacity of the platinum-silver, which was composed of 2 parts by weight of silver and 1 part of platinum, is 0·04877 between 0° C. and 100° C., and the observed value is 0·04864. Again, the calculated thermal capacity of the German-silver, which was composed of 74 parts by weight of copper, 20 parts by weight of zinc, and 6 parts of nickel,† is 0·09460, and the observed value 0·09466. Such results as these are highly satisfactory, as they not only serve as indications of the trustworthiness of the mode of observation, but also show how very closely we may rely on calculating the thermal capacities of such alloys as these from the thermal capacities of their component parts.

Finally, we learn from Table II how well the observations of different observers agree with each other, so that small differences in the purity of the metal‡ do not, as in the case of electrical conductivity, make much difference in the thermal capacity.

Relation between "Young's Modulus" and Thermal Capacity per Unit Volume.

It is now time to consider whether such a relationship as has been mentioned in the beginning of this portion of the paper to be likely to exist between thermal capacity and elasticity, does really do so, and in Table III will be found the necessary data.

From Table III we learn that, with exceptions in the cases of platinum and platinum-silver, the order of the thermal capacities per unit volume of the different metals is the same as the order of

* The numbers given in this column are taken from Everett's "Units and Physical Constants," p. 83, with the exception of those taken from Table I.

† The component parts of this alloy were determined for me in the Chemical Laboratory, at King's College, through the courtesy of Mr. J. M. Thomson.

‡ The metals aluminium, copper, platinum, and silver were sold to me by Johnson, Matthey, and Co. as chemically pure, whilst the zinc, lead, and tin were as pure as could be obtained by the ordinary process of distillation.

Table III.

Metal.	Density at 20° C. Density of water at 4° C. = 1.	Thermal capacity per unit volume at 20° C. Thermal capacity of water at 0° C. = 1. C_v.	$\sqrt[3]{C_v} = \frac{1}{\alpha}$ where α is proportional to the mean distance between the centres of consecutive molecules.	"Young's modulus" in grams per sq. cm. e.	$\frac{e}{C_v^{\frac{2}{3}}}$. $= e \times \alpha^2$.
Iron (1)	7·759	·8443	·9452	1981×10^6	2941×10^6
Copper (2)....	8·851	·8087	·9315	1218	2001
German-silver (2)	8·632	·8010	·9289	1230	2060
Platinum (1)..	21·309	·6869	·8824	1467	3521
Zinc (1).....	7·138	·6537	·8680	766·9	2067
Platinum-silver (1).....	12·616	·6031	·8449	1044	3397
Silver (1)	10·464	·5811	·8344	753·4	2674
Aluminium (1)	2·731	·5780	·8329	671·2	2414
Tin (1)	7·264	·3904	·7308	277·1	2489
Lead (1)	11·193	·3424	·6995	167·0	2038
				Mean	2561×10^6

"Young's Modulus," whilst it will be seen from the last column of the table that the products of e and $\frac{1}{C_v^{\frac{2}{3}}}$ all lie between 20×10^8 and 30×10^8, except as before in the case of platinum and platinum-silver. It seemed advisable therefore to ascertain how far the constancy holds good in the case of such other metals as have been examined by other observers, for the purpose of determining the values of "Young's Modulus" and the thermal capacities per unit volume. Table III is accordingly supplemented by Table IV.

Table IV.

Metal.*	Density.	Thermal capacity per unit volume between 0° C. and 100° C. C_v†.	"Young's modulus." e.	$\frac{e}{C_v^{\frac{7}{3}}}$.
Cadmium .	8·665	·4913	575×10^6	3017×10^6
Glass	2·942	·5207	614·4	2819
Gold	18·275	·5920	685·5	2330
Brass.....	8·471	·7958	1005	1714
Palladium.	11·288	·6694	1077	2747
Piano-steel.	7·727	·8748	2049	2801
Nickel	8·705	·9559	2480	2755
			Mean	2597×10^6

Of the seven substances entered in Table IV, we find five for which the values of $\frac{e}{C_v^{\frac{7}{3}}}$ lie closely to the mean value 2597×10^6, whilst brass, an alloy be it remembered of copper and zinc, furnishes an exceptionally low value. The mean value here agrees fairly with the mean value recorded in Table III, namely, 2561×10^6. But even in some of those cases where the departure from the mean value is great, more recent investigations of my own on "Young's Modulus" and on torsional rigidity, in which very great precautions have been taken to avoid the effects of imperfect elasticity, have resulted in giving values of e which would make the products of e and $\frac{1}{C_v^{\frac{7}{3}}}$ lie closer to the mean values here obtained. This is so for the metals copper, zinc, and brass, but not for platinum, which is still remarkable for having the value of $\frac{e}{C_v^{\frac{7}{3}}}$ much higher than the mean. It may be remarked that the ratio of lateral contraction to longitudinal extension was, according to my previous determinations, *much less* with platinum than with any of the other metals.‡

* The numbers in columns 1 and 4 are, with an exception in the case of nickel, taken from Sir W. Thomson's article on Electricity, "Encyc. Brit." For the elasticities of nickel see "Phil. Trans.," Part I, 1883, p. 126.

† As there are no data for determining the thermal capacity at 20° C., that between 0° C. and 100° C. is here given: therefore C_v is here slightly higher than it should be if we are to compare the results given in the two tables.

‡ *Loc. cit.*, p. 28.

FIG. 1.

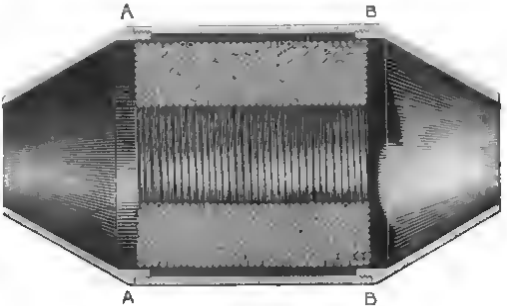


FIG. 2.